NEET | IIT-JEE | FOUNDATION

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JEE MAINS-2018

15-04-2018 Online (Evening)

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with only Black Ball Point Pen provided in the examination hall.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of 3 hours duration.
4. The Test Booklet consists of 90 questions. The maximum marks are 360.
5. There are three parts in the question paper A, B, C consisting of **Chemistry, Mathematics and Physics** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for each correct response.
6. Candidates will be awarded marks as started above in instruction No. 5 for correct response of each question. ¼ (one fourth) marks of the total marks allotted to the question (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from that total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. For writing particulars / marking responses on Side–1 and Side–2 of the Answer Sheet use only Black Ball Point Pen provided in the examination hall.
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in four pages at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. However, the candidates are allowed to take away this Test Booklet with them.
12. The CODE for this Booklet is D. Make sure that the CODE printed on Side–2 of the Answer Sheet is same as that on this Booklet. Also tally the serial number of Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet

PART-A-CHEMISTRY

1. $\Delta_f G°$ at 500 K for substance 'S' in liquid state and gaseous state are +100.7 kcal mol^{-1} and +103 kcal mol^{-1}, respectively. Vapour pressure of liquid 'S' at 500 K is approximately equal to :
[R = 2 cal K^{-1} mol^{-1}]

(1) 100 atm (2) 1 atm (3) 10 atm (4*) 0.1 atm

Sol. $S(\ell) \rightarrow S(g)$

$\Delta G° = \Delta G°_{f,s(g)} - \Delta G°_{f,s(\ell)}$

$= 130 - 100.7 = 2.3$ K Cal

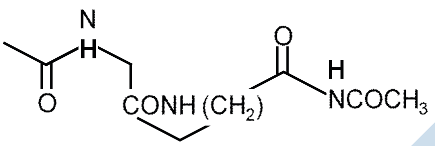
$\Delta G° = -RT \ln P$

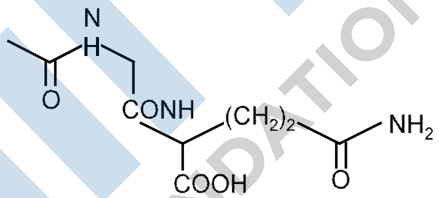
$2.3 \times 1000 = -2.3 \times 2 \times 500 \log P$

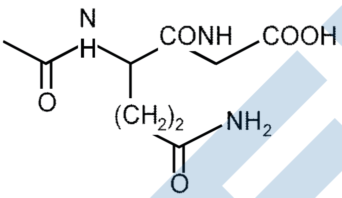
Log P = – 1

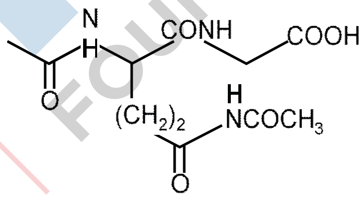
P = 0.1 atm

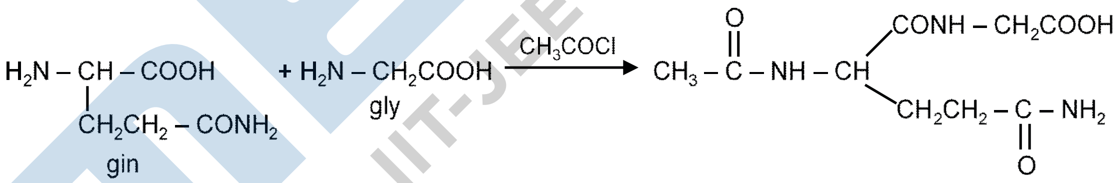
2. The dipeptide, Gln-Gly, on treatment with CH_3COCl followed by aqueous work up gives:

(1) 

(2) 

(3*) 

(4) 

Sol. 

3. For a first order reaction, $A \rightarrow P$, $t_{1/2}$ (half-life) is 10 days. The time required for $\frac{1}{4}$th conversion of A (in days) is : [ln 2 = 0.693, ln 3 = 1.1]

(1*) 4.1 (2) 2.5 (3) 5 (4) 3.2

Sol. $A \longrightarrow P$

For first order $\quad t_{1/2} = \frac{\ln 2}{k} = 10$ days

for $\quad t_{1/4} = \frac{\ln 4/3}{k} = t$

$\Rightarrow \frac{\ln 2}{2\ln 2 - \ln 3} = \frac{10}{t}$

$$t_{1/4} = \frac{(2 \times 0.693 - 1.1)}{0.693} \times 10 = 4.1$$

4. The correct order of electron affinity is :

(1*) Cl > F > O (2) O > F > Cl (3) F > Cl > O (4) F > O > Cl

Sol. The correct order of electron affinity is Cl > F > O.

5. In KO_2, the nature of oxygen species and the oxidation state of oxygen atom are, respectively:

(1) Peroxide and –1/2 (2) Superoxide and –1

(3*) Superoxide and –1/2 (4) Oxide and –2

Sol. KO_2 Potassium superoxide

$K^+O_2^{-1}$; Oxidation sate of oxygen in O_2^{-1} is $\frac{-1}{2}$.

6. Biochemical Oxygen Demand (BOD) value can be a measure of water pollution caused by the organic matter. Which of the following statements is correct?

(1) Anaerobic bacteria increase the BOD value

(2) Aerobic bacteria decrease the BOD value

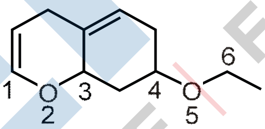
(3*) Polluted water has BOD value higher than 10 ppm.

(4) Clean water has BOD value higher than 10 ppm.

Sol. Polluted water has BOD value higher than 10 ppm.

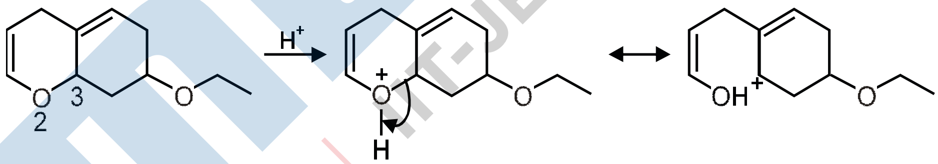
Clean water has less than 5 ppm.

7. On treatment of the following compound with a strong acid, the most susceptible site for bond cleavage is :

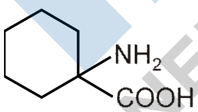


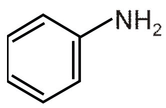
(1*) O2 – C3 (2) C4 – O5 (3) C1 – O2 (4) O5 – C6

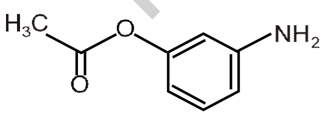
Sol.

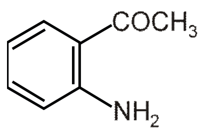


8. The increasing order of diazotisation of the following compound is :

(a) 

(b) 

(c) 

(d) 

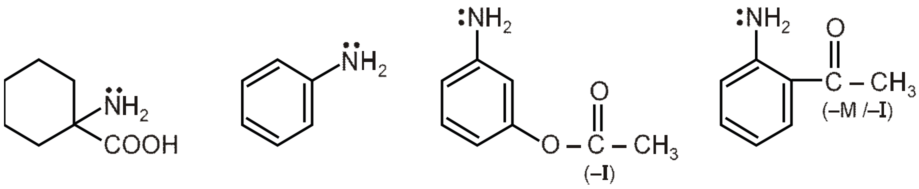
(1) (a) < (d) < (b) < (c) (2) (a) < (d) < (c) < (b)

(3) (a) < (b) < (c) < (d) (4*) (d) < (c) < (b) < (a)

Sol. $Ph/R - \ddot{N}H_2 + \overset{\oplus}{N}=O \longrightarrow Ph/R - NH - N \longrightarrow Ph/R - N_2^{\oplus}$

In diazotisation, aliphatic amine or aromatic amine act as nucleophile or base

BaHe strength or nucleophilicity of amine increases, then role of diazotization increases.



Base strength or nucleophilicity ↑

Role of diazotisation ↑

9. When 2-butyne is treated with H_2/ Lindlar's catalyst, compound X is produced as the major product and when treated with Na/liq. NH_3 it produces Y as the major product. Which of the following statements is correct ?

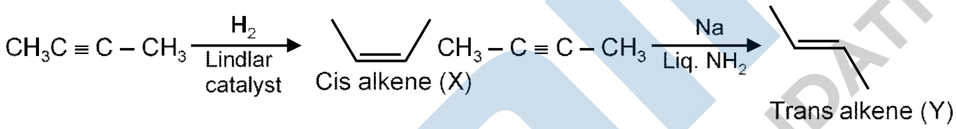
(1*) X will have higher dipole moment and higher boiling point than Y.

(2) X will have lower dipole moment and lower boiling point than Y.

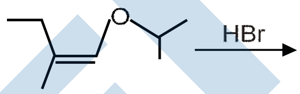
(3) Y will have higher dipole moment and lower boiling point than X.

(4) Y will have higher dipole moment and higher boiling point than X.

Sol.

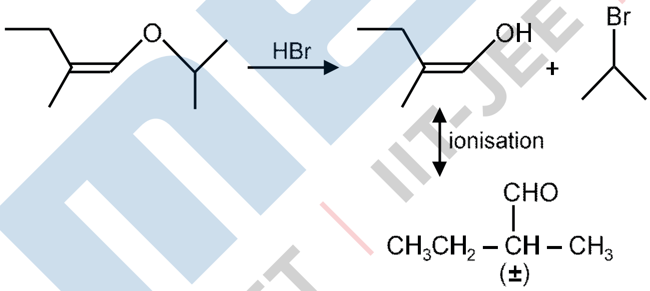


10. The total number of optically active compounds formed in the following reaction is :



(1) Zero (2) Six (3*) Four (4) Two

Sol.



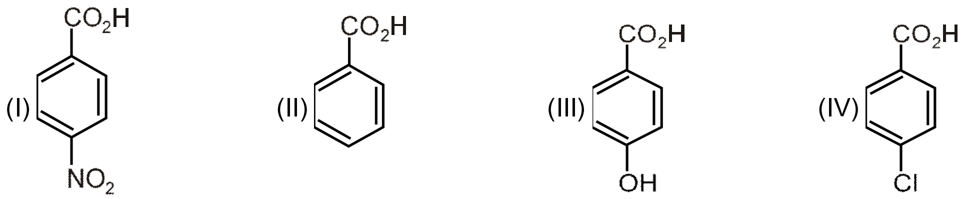
11. Two 5 molal solutions are prepared by dissolving a non-electrolyte non-volatile solute separately in the solvents X and Y. The molecular weights of the solvents are M_X and M_Y, respectively where $M_X = \frac{3}{4} M_Y$. The relative lowering of vapour pressure of the solution in X is 'm' times that of the solution in Y. Given that the number of moles of solute is very small in comparison to that of solvent, the value of 'm' is :

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3*) $\frac{3}{4}$ (4) $\frac{4}{3}$

Sol. RLVP $\propto$ Molar mass of solvent .....(If molality is same)

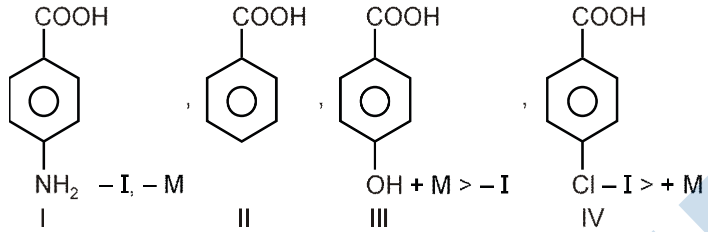
$$\frac{(RLVP)_X}{(RLVP)_Y} = \frac{M_X}{M_Y} = \frac{3}{4}$$

12. The increasing order of the acidity of the following carboxylic acids is :



(1) II < IV < III < I   (2) IV < II < III < I   (3) I < III < II < IV   (4*) III < II < IV < I

Sol.



Acidic strength I > IV > II > III

13. In the leaching method, bauxite ore is digested with a concentrated solution of NaOH that produces 'X'. When CO_2 gas is passed through the aqueous solution of 'X' , a hydrated compound 'Y' is precipitated. 'X' and 'Y' respectively are :

(1) $NaAlO_2$ ansd $Al_2(CO_3)_3 \cdot xH_2O$ (2) $Al(OH)_3$ and $Al_2O_3 \cdot xH_2O$

(3) $Na[Al(OH)_4]$ and $Al_2(CO_3)_3 \cdot xH_2O$ (4*) $Na[Al(OH)_4]$ and $Al_2O_3 \cdot xH_2O$

Sol. $$Al_2O_3 + 2NaOH + 3H_2O \longrightarrow \underset{(x)}{2Na[Al(OH)_4]} \xrightarrow{CO_2} Al_2O_3 \cdot xH_2O$$

14. All of the following share the same crystal structure except:

(1*) LiCl   (2) NaCl   (3) CsCl   (4) RbCl

Sol. CsCl has Bcc structure.

Cs ions form the simple cubic arrangement Cl^- ion occupy the cubic interstitial sites, each Cl^- ion has eight Cs^+ as its nearest neighbours.

15. Lithium aluminium hydride reacts with silicon tetrachloride to form:

(1) LiH, $AlCl_3$ and $SiCl_2$ (2) LiCl, AlH_3 and SiH_4

(3) LiGH, AlH_3 and SiH_4 (4*) LiCl, $AlCl_3$ and SiH_4

Sol. $LiAlH_4 \longrightarrow LiH + AlH_3$

$4LiH + SiCl_4 \longrightarrow SiH_4 + 4LiCl$

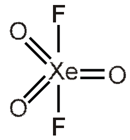
$4AlH_3 + 3SiCl_4 \longrightarrow 4AlCl_3 + 3SiH_4$

16. In XeO_3F_2, the number of bond pair(s), π-bond(s) and lone pair(s) on Xe atom respectively are:

(1) 5, 2, 0   (2) 4, 2, 2   (3) 4, 4, 0   (4*) 5, 3, 0

Sol. XeO_3F_2

SN = 10, sp^3d, Trigonal bipyramidal



Number of bond pair = 5

Number of π bond = 3

Lone pair =0

17. Given :

(i) $2Fe_2O_3\,(s) \longrightarrow 4Fe(s) + 3O_2\,(g)$; $\Delta_rG° = +\,1487.0$ kJ mol^{-1}

(ii) $2CO(g) + O_2\,(g) \longrightarrow 2CO_2\,(g)$; $\Delta_rG° = -\,514.4$ kJ mol^{-1}

Free energy change, $\Delta_rG°$ for the reaction $2Fe_2O_3\,(s) + 6CO(g) \longrightarrow 4Fe(s) + 6CO_2\,(g)$ will be

(1*) – 56.2 kJ mol^{-1} (2) –208.0 kJ mol^{-1} (3) –112.4 kJ mol^{-1} (4) –168.2 kJ mol^{-1}

Sol. $2Fe_2O_3\,(s) \longrightarrow 4Fe(s) + 3O_2\,(g)$; $\Delta_rG° = +\,1487.0$ kJ mol^{-1}

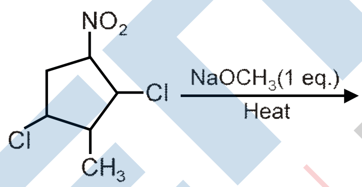
$2CO(g) + O_2\,(g) \longrightarrow 2CO_2\,(g)$; $\Delta_rG° = -\,514.4$ kJ mol^{-1}

For reaction

$2Fe_2O_3 + 6CO \longrightarrow 4Fe + 6CO_2$

$\Delta G = -\,514.4 \times 3 + 1487 = -\,56.2$ kJ mol^{-1}

18. The major product formed in the following reaction is :



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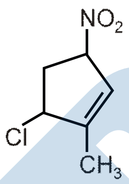

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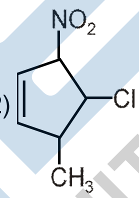

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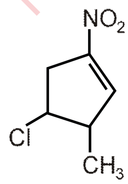

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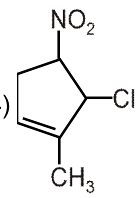
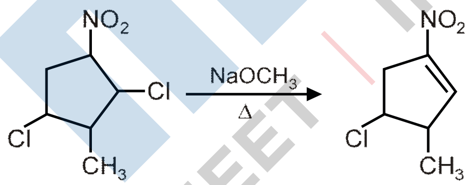


Sol.


19. The correct order of spin-only magnetic moments among the following is :

[Atomic number : Mn = 25, Co = 27, Ni = 28, Zn = 30]

(1) $[NiCl_4]^{2-} > [CoCl_4]^{2-} > [MnCl_4]^{2-} > [ZnCl_4]^{2-}$

(2*) $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$

(3) $[CoCl_4]^{2-} > [MnCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$

(4) $[ZnCl_4]^{2-} > [NiCl_4]^{2-} > [CoCl_4]^{2-} > [MnCl_4]^{2-}$

Sol. $[MnCl_4]^{-2}$; 0.5 of Mn is +2

Mn^{+2} ; $3d^5$, 5 unpaired electron

Co^{+2} ; $3d^7$, 3 unpaired electron

$Zn^{+2} \rightarrow 3d^{10}$, 0 unpaired electron

$Ni^{+2} \rightarrow 3d^8$, 2 unpaired electron

Spin magnetic moment

$\mu = \sqrt{n(n+2)}$

n $\rightarrow$ Number of unpaired electron

Correct order of spin only magnetic moments : $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$

20. Following four solutions are prepared by mixing different volumes of NaOH and HCl of different concentrations, pH of which one of them will be equal to 1?

(1) 55 mL $\frac{M}{10}$HCl + 45 ml $\frac{M}{10}$NaOH

(2*) 75 mL $\frac{M}{5}$HCl + 25 mL $\frac{M}{5}$ NaOH

(3) 60 mL $\frac{M}{10}$ HCl + 40 mL $\frac{M}{10}$NaOH

(4) 100 mL $\frac{M}{10}$HCl + 100 mL $\frac{M}{10}$ NaOH

Sol. $HCl + NaOH \longrightarrow H_2O + NaCl$

(i) $[H^+] = \frac{10}{100}$ $\Rightarrow$ pH = 1

(ii) $[H^+] = [OH^-] = 10^{-7}$ $\Rightarrow$ pH = 7 Neutral

(iii) $[H^+] = \frac{2}{100}$ $\Rightarrow$ pH = 2 – log2 = 2 – 0.3 = 1.7

(iv) $[H^+] = \frac{1}{100}$ $\Rightarrow$ pH = 2

21. At a certain temperature in a 5 L vessel, 2 moles of carbon monoxide and 3 moles of chlorine were allowed to reach equilibrium according to the reaction,

$$CO + Cl_2 \rightleftharpoons COCl_2$$

At equilibrium, if one mole of CO is present then equilibrium constant (K_C) for the reaction is :

(1) 3 (2*) 2.5 (3) 2 (4) 4

Sol.

	CO	+ Cl_2	$\rightleftharpoons$ $COCl_2$
t = 0	2	3	
t = t	2 – x	3 – x	x

Moles of CO = 2 – x = 1

x = 1

$$K_C = \frac{\left(\frac{x}{v}\right)}{\left(\frac{2-x}{v}\right)\left(\frac{3-x}{v}\right)} = \frac{5}{2} = 2.5$$

22. The de-Broglie's wavelength of electron present in first Bohr orbit of 'H' atom is :

(1*) $2\pi \times 0.529$ Å (2) 0.529 Å (3) $\frac{0.529}{2\pi}$ Å (4) 4×0.529 Å

Sol. $mvr = n\frac{h}{2\pi}$

$2\pi r = n\left(\frac{h}{mv}\right)$

$= n\lambda$

$\lambda \rightarrow$ de-Broglie's wavelength

So for first Bohr orbit wavelength of electron = $2\pi r_1$

$= 2\pi(0.529)$ Å

23. The total number of possible isomers for square-planar $[Pt(Cl)(NO_2)(NO_3)(SCN)]^{2-}$ is :

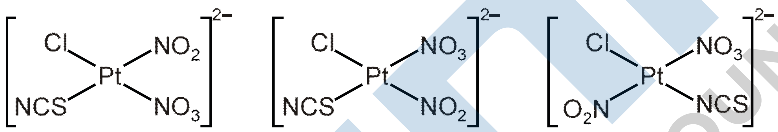
(1) 24 (2*) 12 (3) 8 (4) 16

Sol. Total number of possible isomers for square planar $[Pt(Cl)(NO_2)(NO_3)(SCN)]^{2-}$.

MABCD complex : 3 geometrical isomers

NO_2 and SCN are ambidentate ligand

total isomers = 4 × 3 = 12



24. Two compound I and II are eluted by column chromatography (adsorption of I > II). Which of the following is a correct statement?

(1*) II moves faster and has higher R_f value than I

(2) I moves faster and has higher R_f value than II

(3) II moves slower and has higher R_f value than I

(4) I moves slower and has higher R_f value than II

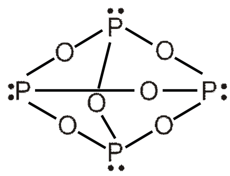
Sol. The principle of column chromatography is based on differential adsorption of substance by the adsorbent. The rate at which the components of a mixture are separated depends on the activity of the adsorbent and polarity of the solvent if the activity of the adsorbent is very high and polarity of the solvent is very low, then the separation is very slow but gives a good separation.

R_f is defined as the ratio of the distance travelled by the center of a spot to the distance travelled by the solvent form greater Q_f, greater affinity of solute to the solvent.

25. The number of P – O bonds in P_4O_6 is :

(1*) 12 (2) 6 (3) 9 (4) 18

Sol. The number of P – O bonds in P_4O_6



26. If x gram of gas is adsorbed by m gram of adsorbent at pressure P, the plot of log $\frac{x}{m}$ versus log P is linear. The slope of the plot is : [n and k are constants and n > 1]

(1) log K (2*) $\frac{1}{n}$ (3) n (4) 2k

Sol. $\frac{x}{m} = kP^{1/n}$

$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$

Y = mx + C

Slope $m = \frac{1}{n}$

27. For per gram of reactant, the maximum quantity of N_2 gas is produced in which of the following thermal decomposition reactions? [Given : Atomic weight: Cr = 52 u, ba = 137 u]

(1) $(NH_4)_2Cr_2O_7(s) \longrightarrow N_2(g) + 4H_2O\ (g) + Cr_2O_3\ (s)$

(2*) $2NH_3(g) \longrightarrow N_2(g) + 3H_2(g)$

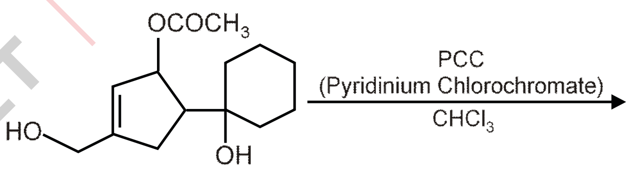
(3) $2NH_4NO_3(s) \longrightarrow 2N_2(g) + 4H_2O(g) + O_2\ (g)$

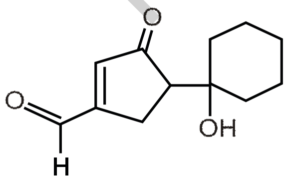
(4) $Na(N_3)_2(s) \longrightarrow Ba(s) + 3N_2\ (g)$

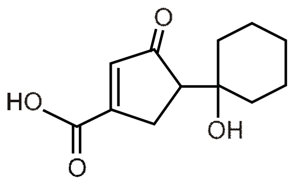
Sol. $2NH_3(g) \longrightarrow N_2(g) + 3H_2(g)$

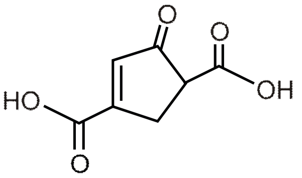
Maximum quantity of N_2 gas produced $= \frac{1}{17} \times \frac{1}{2} \times 28 = 0.82$

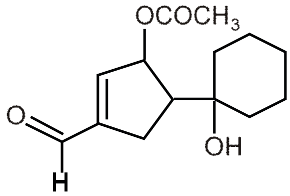
28. The major product formed in the following reaction is :

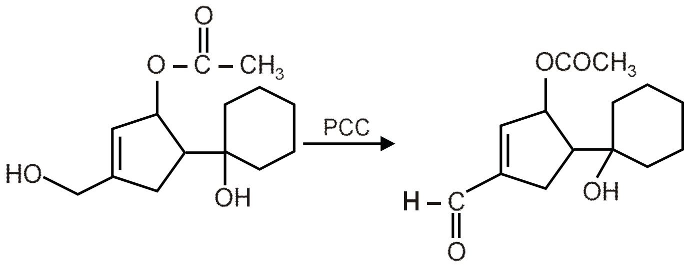


(1) 

(2) 

(3) 

(4*) 

Sol. 

29. Which of the following statement is not true ?

(1) Chain growth polymerisation includes both homopolymerisation and copolymerisation

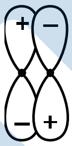
(2*) Chain growth polymerisation involves homopolymerisation only.

(3) Step growth polymerisation requires a bifunctional monomer.

(4) Nylon 6 is an example of step-growth polymerisation.

Sol. Chain growth polymerization can involve homopolymerisation and with as copolymerisation.

30. Which of the following best describes the diagram below of a molecular orbital ?

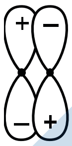


(1) An antibonding σ orbital

(2) A non-bonding orbital

(3) A bonding π orbital

(4*) An antibonding π orbital

Sol. 

An antibonding π orbital

PART-B-MATHEMATICS

31. If $I_1 = \int_0^1 e^{-x}\cos^2 x\,dx$, $I_2 = \int_0^1 e^{-x^2}\cos^2 x\,dx$ and $I_3 = \int_0^1 e^{-x^3}dx$, then

(1) $I_2 > I_1 > I_3$ (2*) $I_3 > I_2 > I_1$ (3) $I_3 > I_1 > I_2$ (4) $I_2 > I_3 > I_1$

Sol. We know that

$x > x^2, \forall\ x \in (0, 1)$

$-x < -x^2$

$\boxed{e^{-x} < e^{-x^2}}$

$e^{-x}\cos^2 x < e^{-x^2}\cos^2 x$

$\int_0^1 e^{-x}\cos^2 x\,dx < \int_0^1 e^{-x^2}\cos^2 x\,dx$

$\boxed{I_1 < I_2}$

Similarly, $x^2 > x^3, \forall\ x \in (0, 1)$

$-x^2 < -x^3$

$e^{-x^2} < e^{-x^3}O$

$\Rightarrow$ $e^{-x^2}\cos^2 x < e^{-x^3}$

$\Rightarrow$ $\int_0^1 e^{-x^2}\cos^2 x\,dx < \int_0^1 e^{-x^3}\,dx$

$I_2 < I_3$

32. A tower T_1 of height 60 m is located exactly opposite to a tower T_2 of height 80 m on a straight road. From the top of T_1, if the angle of depression of the foot of T_2 is twice the angle of elevation of the top of T_2, then the width (in m) of the road between the feet of the towers T_1 and T_2 is

(1) $10\sqrt{3}$ (2) $10\sqrt{2}$ (3*) $20\sqrt{3}$ (4) $20\sqrt{2}$

Sol. In ΔABC In ΔADE

$\tan 2\alpha = \frac{60}{x}$ ...(1) $\frac{20}{x} = \tan\alpha$ ...(1)

From (1) and (2)

$$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{60}{x}$$

$$\Rightarrow \frac{2\times\frac{20}{x}}{1-\frac{400}{x^2}} = \frac{60}{x} \Rightarrow \boxed{x = 20\sqrt{3}}$$

33. The value of integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}\,dx$ is

(1) $\pi\sqrt{2}$ (2) $\frac{\pi}{2}\left(\sqrt{2}+1\right)$ (3*) $\pi\left(\sqrt{2}-1\right)$ (4) $2\pi\left(\sqrt{2}-1\right)$

Sol. $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}\,dx$ ...(1)

Use KING,

$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi - x}{1+\sin x}\,dx$ ...(2)

(1) + (2)

$$2I = \pi\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\sin x}$$

$$2I = \pi\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\frac{1-\sin x}{\cos^2 x}\right)$$

$$2I = \pi(\tan x - \sec x)_{\pi/4}^{3\pi/4}$$

$$\Rightarrow \quad I = \frac{\pi}{2}\left(-1-1-\left(-\sqrt{2}-\sqrt{2}\right)\right) = \left(\sqrt{2}-1\right)\pi$$

34. If a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. such that a < b < c and a + b + c = $\frac{3}{4}$,then the value of a is

(1) $\frac{1}{4}-\frac{1}{4\sqrt{2}}$ (2) $\frac{1}{4}-\frac{1}{3\sqrt{2}}$ (3*) $\frac{1}{4}-\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{4}-\frac{1}{\sqrt{2}}$

Sol. Given 2b = a + c ....(1)

$b^4 = a^2c^2$

$b^2 = ac$ ....(2)

$a + b + c = \frac{3}{4}$

$3b = \frac{3}{4}$

$\boxed{b = \frac{1}{4}}$ ....(3)

Put in (1) $a + c = \frac{1}{2}$ ....(4)

Put in (2) $ac = \frac{1}{16}$

$$2ac = \frac{1}{8}$$

$$(a-c)^2 = (a+c)^2 - 4ac = \frac{1}{4} - \frac{4}{32} = \frac{4}{32} = \frac{1}{8}$$

$$a - c = \frac{-1}{2\sqrt{2}} \quad ....(5) \quad (\because\ a < c)$$

add (4) + (5)

$$2a = \frac{1}{2} - \frac{2}{2\sqrt{2}}$$

$$\boxed{a = \frac{1}{4} - \frac{1}{2\sqrt{2}}}$$

35. Tangents drawn from the point (– 8, 0) to the parabola $y^2 = 8x$ touch the parabola at P and Q. If F is the focus of the parabola, then the area of the triangle PFQ (in sq. units) is equal to

(1) 64 (2) 32 (3) 24 (4*) 48

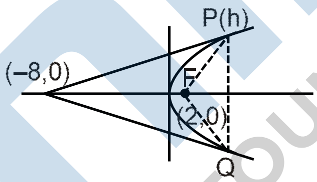
Sol. $ty = x + 2t^2$

Passes from (–8, 0)

$\Rightarrow \quad t^2 = 4$

$t = \pm 2$

P(8, 8), Q(8, –8)



Sol. -2 Equation of tangent at parabola

$$y = mx + \frac{2}{m} \qquad (-8, 0)$$

$$0 = -8m + \frac{2}{m} \qquad \text{Slope of tangent} = \frac{1}{t}$$

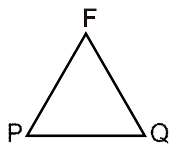
$$m^2 = \frac{1}{4} \qquad \Rightarrow \boxed{t = \pm 2}$$

$$\boxed{m = \pm\frac{1}{2}}$$

$P(2 \times 2^2, 2 \times 2 \times 2)$

P(8, 8) and (8, –8)

F(2, 0) $\quad A_r = \frac{1}{2} \times 6 \times 16 = 48$



36. A player X has a biased coin whose probability of showing heads is p and a player Y has a fair coin. They start playing a game with their own coins and play alternately. The player who throws a head first is a winner. If X starts the game, and the probability of winning the game by both the players is equal, then the value of p is

(1) $\frac{1}{5}$ (2) $\frac{1}{4}$ (3) $\frac{2}{5}$ (4*) $\frac{1}{3}$

Sol. P (X win) = $\frac{1}{2}$

$$X+\bar{X}\bar{Y}X+\overline{XYXY}X+.....=\frac{1}{2}$$

$$P+(1-P)\frac{1}{2}\cdot P+\left((1-P)\frac{1}{2}\right)^2 P+.....=\frac{1}{2}$$

$$\frac{P}{1-\frac{(1-P)}{2}}=\frac{1}{2}$$

$\Rightarrow 2P=\frac{2-(1-P)}{2}$ $\Rightarrow$ 4P = 1 + P

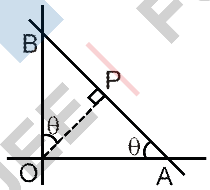
$$P=\frac{1}{3}$$

37. The foot of the perpendicular drawn from the origin, on the line, 3x + y = λ ($\lambda \neq 0$) is P. If the line meets x-axis at A and y-axis at B, then the ratio BP : PA is

(1*) 9 : 1 (2) 3 : 1 (3) 1 : 9 (4) 1 : 3

Sol. In ΔAOP

$$\tan\theta = \frac{OP}{AP}$$



In ΔBOP

$$\tan\theta = \frac{BP}{OP}$$

$$\Rightarrow \frac{BP}{AP}=\tan^2\theta=\frac{9}{1}$$

38. If f (x) = $\sin^{-1}\left(\frac{2\times 3^x}{1+9^x}\right)$, then $f'\left(-\frac{1}{2}\right)$ equals

(1*) $\sqrt{3}\log_e\sqrt{3}$ (2) $-\sqrt{3}\log_e\sqrt{3}$ (3) $\sqrt{3}\log_e 3$ (4) $-\sqrt{3}\log_e 3$

Sol. Put $3^x = \tan\theta$

$$\forall x=\frac{-1}{2},\ 2\theta\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$$

So, $f(x) = \sin^{-1}\sin 2\theta$

$f(x) = 2\tan^{-1}3^x$

$$f'(x)=\frac{2\cdot 3^x \ln 3}{1+9^x}$$

$$f'\left(\frac{-1}{2}\right)=\sqrt{3}\ln\sqrt{3}$$

39. Let f : A → B be a function defined as $f(x)=\frac{x-1}{x-2}$, where A = R – {2} and B = R – {1}. Then f is

(1) invertible and $f^{-1}(y)=\frac{3y-1}{y-1}$ (2) not invertible

(3*) invertible and $f^{-1}(y)=\frac{2y-1}{y-1}$ (4) invertible and $f^{-1}(y)=\frac{2y+1}{y-1}$

Sol. $f(x)=\frac{x-1}{x-2}$ a

$f'(x)=\frac{-1}{(x-2)^2}<0$, f is one-one

and $y=\frac{x-1}{x-2} \Rightarrow x=\frac{2y-1}{y-1}$

Range = R – {1}so, f is onto

and $f^{-1}(x)=\frac{2x-1}{x-1}$

40. Let $f(x)=\begin{cases}(x-1)^{\frac{1}{2-x}}, & x>1, x\neq 2\\ k, & x=2\end{cases}$

The value of k for which f is continuous at x = 2 is

(1*) e^{-1} (2) 1 (3) e^{-2} (4) e

Sol. f will be continuous if

$$\lim_{x\to 2}(x-1)^{\frac{1}{2-x}}=k$$

$$k=\lim_{x\to 2}\frac{1}{(2-x)}((x-1)-1)=e^{-1}$$

41. If f (x) is a quadratic expression such that f (1) + f (2) = 0 and – 1 is a root of f (x) = 0, then the other root of f (x) = 0 is

(1*) $\frac{8}{5}$ (2) $-\frac{8}{5}$ (3) $-\frac{5}{8}$ (4) $\frac{5}{8}$

Sol. Let $f(x)=a(x+1)(x-\alpha)$

$\Rightarrow$ f (1) + f (2)

$\Rightarrow$ $a(2)(1-\alpha)+a(3)(2-\alpha)=0$

$\Rightarrow \quad 2 - 2\alpha + 6 - 3\alpha = 0$

$\Rightarrow \quad \alpha = \frac{8}{5}$

42. Let f (x) be a polynomial of degree 4 having extreme values at x = 1 and x = 2. If $\lim_{x\to 0}\left(\frac{f(x)}{x^2}+1\right) = 3$ then f (–1) is equal to

(1) $\frac{5}{2}$ (2*) $\frac{9}{2}$ (3) $\frac{3}{2}$ (4) $\frac{1}{2}$

Sol. f '(1) = f '(2) = 0

$\boxed{f(0)=0}$ repeat root and for limit exist coefficient of x^2 power is 2.

$f'(x) = k(x-1)(x-2)x$

$f'(x) = k(x^3 - 3x^2 + 2x)$

$f(x) = k\left(\frac{x^4}{4} - x^3 + x^2\right) + c$

$\because \quad f(0) = 0 \Rightarrow c = 0$

and coefficient of $x^2 = 2$

$\boxed{k=2}$

$f(x) = \frac{x^4}{2} - 2x^3 - 2x^2$

$f(-1) = \frac{1}{2} - 2 + 2 = \frac{1}{2}$

43. The coefficient of x^{10} in the expansion of $(1 + x)^2 (1 + x^2)^3 (1 + x^3)^4$ is equal to

(1) 50 (2) 44 (3) 56 (4*) 52

Sol. ${}^2C_p x^p \cdot {}^3C_q \cdot x^{2z} \cdot {}^4C_r \cdot x^{3r}$

$p + 2q + 3r = 10, \quad 0 \le p \le 2, \quad 0 \le q \le 3, \quad 0 \le r \le 4$

$r = 3, \quad q = 0, \quad p = 1$

$\Rightarrow \quad {}^2C_1 \cdot {}^3C_0 \cdot {}^4C_3 = 8$

$r = 2, \quad q = 2, \quad p = 0 \quad \Rightarrow \quad {}^2C_0 \cdot {}^3C_2 \cdot {}^4C_2 = 18$

$r = 2, \quad q = 1, \quad p = 2 \quad \Rightarrow \quad {}^2C_2 \cdot {}^3C_1 \cdot {}^4C_2 = 18$

$r = 1, \quad q = 3, \quad p = 1 \quad \Rightarrow \quad {}^2C_1 \cdot {}^3C_3 \cdot {}^4C_1 = 8$

44. If $|z - 3 + 2i| \le 4$, then the difference between the greatest value and the least value of |z| is

(1*) $2\sqrt{13}$ (2) 8 (3) $\sqrt{13}$ (4) $4+\sqrt{13}$

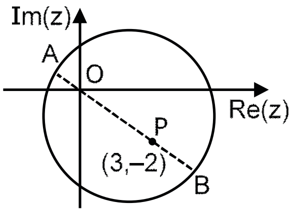
Sol. $|z - 3 + 2i| \le 4$ represent

Circle with center (3, –2), rad = 4

$|z| = OB - OA$, $OP = \sqrt{13}$

$= (OP + r) - (r - OP)$

$= 2(OP) = 2\sqrt{13}$



45. $\lim\limits_{x\to 0} \dfrac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ equals

(1) $-\dfrac{1}{2}$ (2) $\dfrac{1}{4}$ (3) 1 (4*) $\dfrac{1}{2}$

Sol. $\lim\limits_{x\to 0} \dfrac{2x\tan x\left(\dfrac{1}{1-\tan^2 x} - 1\right)}{4\sin^4 x}$

$\lim\limits_{x\to 0} \dfrac{2x\tan x\left(\dfrac{\tan^2 x}{1-\tan^2 x}\right)}{4\sin^4 x} = \dfrac{1}{2}$

46. The number of four letter words that can be formed using the letters of the word BARRACK is

(1) 144 (2) 120 (3) 264 (4*) 270

Sol. BARRACK

Category	Selection	Arrangement
4 different	${}^5C_4$	${}^5C_4 \times 4!$
2 different, 2 Alike	${}^2C_1 \times {}^4C_2$	${}^2C_1 \times {}^4C_2 \times \dfrac{4!}{2!}$
2 Alike, 2 other alike	1	$\dfrac{4!}{2!\,2!}$

Total word = 120 + 144 + 6 = 270

47. The number of solutions of sin 3x = cos 2x, in the interval $\left(\dfrac{\pi}{2}, \pi\right)$ is

(1) 3 (2) 2 (3) 4 (4*) 1

Sol. cos 2x = sin 3x

cos 2x = cos(90 – 3x)

$2x = 2x\pi \pm (90 - 3x)$

Take positive Take negative

$5x = 2n\pi + \dfrac{\pi}{2}$ $x = -2x\pi + \dfrac{\pi}{2}$

$n = 1, x = \dfrac{\pi}{2}$ (reject) $n \in I$

$n = 2,\ x = \dfrac{9\pi}{10}$ No solution

$n = 3,\ x = \dfrac{13\pi}{10}$ (reject) in $\left(\dfrac{\pi}{2}, \pi\right)$

So, one solution

48. Consider the following two statements:

Statement p : The value of sin 120º can be derived by taking θ = 240º in the equation

$$2\sin\frac{\theta}{2} = \sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}\ .$$

Statement q : The angles A, B, C and D of any quadrilateral ABCD satisfy the equation

$$\cos\left(\frac{1}{2}(A+C)\right) + \cos\left(\frac{1}{2}(B+D)\right) = 0$$

Then the truth values of p and q are respectively

(1) F, F (2) T, F (3*) F, T (4) T, T

Sol. Statement q : $\cos\dfrac{A+C}{2} + \cos\dfrac{B+D}{2}$

$\because \quad 2\cos\left(\dfrac{A+B+C+D}{4}\right)\cos\left(\dfrac{A+C-B-D}{4}\right) = 0$

If $\quad A + B + C + D = 2\pi$

$\Rightarrow \quad \cos\left(\dfrac{A+B+C+D}{4}\right) = 0$

Statement q is true.

Statement p : $\sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}$

$$= \sqrt{\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)^2} - \sqrt{\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)^2}$$

$$= \left|\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right| - \left|\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right|$$

$$\cos\frac{\theta}{2} + \sin\frac{\theta}{2} - \left(\sin\frac{\theta}{2} - \cos\frac{\theta}{2}\right) \qquad \left\{\because \sin\frac{\theta}{2} > \cos\frac{\theta}{2}\right\}$$

$$= 2\cos\frac{\theta}{2} \text{ (false)}$$

49. If the system of linear equations

x + ay + z = 3

x + 2y + 2z = 6

x + 5y + 3z = b

has no solution, then

(1) $a \neq -1$, b = 9 (2*) a = –1, $b \neq 9$ (3) a = –1, b = 9 (4) a = 1, $b \neq 9$

Sol. For no solution, D = 0 $\Rightarrow$ $\begin{vmatrix} 1 & a & 1 \\ 1 & 2 & 2 \\ 1 & 5 & 3 \end{vmatrix} = 0$

$\Rightarrow$ 6 + 5 + 2a – 2 – 10 – 3a = 0

$\boxed{a = -1}$

and $D_x \neq 0$, $\begin{vmatrix} 3 & a & 1 \\ 6 & 2 & 2 \\ b & 5 & 3 \end{vmatrix} \neq 0$

18 + 30 + 2ab – 2b – 30 – 18a = 0

4b ≠ 36 $\Rightarrow$ $\boxed{b \neq 9}$

50. An angle between the lines whose direction cosines are given by the equations,

ℓ + 3m + 5n = 0 and 5lm – 2mn + 6nl = 0, is

(1) $\cos^{-1}\left(\frac{1}{3}\right)$ (2*) $\cos^{-1}\left(\frac{1}{6}\right)$ (3) $\cos^{-1}\left(\frac{1}{4}\right)$ (4) $\cos^{-1}\left(\frac{1}{8}\right)$

Sol. Put ℓ = –3m – 5n in II expression ....(1)

5(–3m – 5n) m – 2mn + 6n (–3m – 5n) = 0

$\Rightarrow$ $-15m^2 - 25nm - 2nm - 18\,mn - 30\,n^2 = 0$

$\Rightarrow$ $15m^2 + 45mn + 30n^2 = 0$

$\Rightarrow$ (m + n) (m + 2n) = 0

$\Rightarrow$ m = –n ....(2)

$\Rightarrow$ m = –2n ....(3)

From (1) and (2) From (1) and (3)

$\frac{\ell}{-2} = \frac{m}{-1} = \frac{n}{1}$ and $\frac{\ell}{1} = \frac{m}{-2} = \frac{n}{1}$

angle $\cos\theta = \frac{-2+2+1}{\sqrt{6}\sqrt{6}} = \frac{1}{6} \Rightarrow \theta = \cos^{-1}\frac{1}{6}$

51. Let $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - .... + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$.

Then, the least odd natural number p, so that $B_n > A_n$, for all n ≥ p, is

(1) 5 (2) 11 (3*) 7 (4) 9

Sol. $A_n = \frac{3}{4} \cdot \frac{1-\left(\frac{-3}{4}\right)^n}{1-\left(\frac{-3}{4}\right)}$

$$= \frac{3}{7}\left(1-\left(\frac{-3}{4}\right)^n\right)$$

Now $\quad B_n > A_n$

$1 > 2A_n$

$A_n < \frac{1}{2}$

$$\frac{3}{7}\left(1-\left(\frac{-3}{4}\right)^n\right) < \frac{1}{2} \Rightarrow \frac{-1}{6} < \left(\frac{-3}{4}\right)^n$$

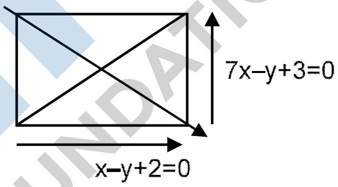
True for all $\quad n \geq 7$

52. The sides of a rhombus ABCD are parallel to the lines, $x - y + 2 = 0$ and $7x - y + 3 = 0$. If the diagonals of the rhombus intersect at P(1, 2) and the vertex A (different from the origin) is on the y-axis, then the ordinate of A is

(1*) $\frac{5}{2}$ (2) $\frac{7}{2}$ (3) $\frac{7}{4}$ (4) 2

Sol. Diagonal are parallel to angle bisector

$$\frac{x-y+2}{\sqrt{2}} = \frac{\pm(7x-y+3)}{5\sqrt{2}}$$



$\Rightarrow \quad 5x - 5y + 10 = \pm(7x - y + 3)$

$2x + 4y - 7 = 0$ or $12x - 6 + 13 = 0$

Diagonal are parallel to angle bisector and passing through (1, 2)

$2x + 4y - 10 = 0,\ 12x - 6y = 0$

$\boxed{x+2y=5}$ $\boxed{2x-y=0}$

So, $\quad A\left(0, \frac{5}{2}\right)$

53. The curve satisfying the differential equation, $(x^2 - y^2)\,dx + 2xydy = 0$ and passing through the point (1, 1) is

(1) an ellipse
(2) a circle of radius two
(3) a hyperbola
(4*) a circle of radius one

Sol. $y^2 = t \quad \Rightarrow \quad 2ydy = dt$

$x^2dx - t \cdot dx + x \cdot dt = 0$

$$\Rightarrow \quad \int \frac{x\,dt - t\,dx}{x^2} = -\int dx$$

$$\Rightarrow \quad \int d\left(\frac{t}{x}\right) = -\int dx$$

$$\Rightarrow \quad \frac{t}{x} = -x + c$$

$\Rightarrow \quad y^2 = -x^2 + 2x$

54. If $\int \frac{2x+5}{\sqrt{7-6x-x^2}}dx = A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right) + C$

(where C is a constant of integration), then the ordered pair (A, B) is equal to

(1) (–2, 1) (2*) (–2, –1) (3) (2, –1) (4) (2, 1)

Sol. 2x + 5 = A(–2x – 6) + B

A = –1, B = –1

$\int \frac{-(2x-6)-1}{\sqrt{7-6x-x^2}} \qquad -\int \frac{-2x-6}{\sqrt{7-6x-x^2}}dx - \int \frac{dx}{\sqrt{7-6x-x^2}}$

$2\sqrt{7-6x-x^2} - \sin^{-1}\frac{x+3}{4} + c$

A = 2, B = –1

55. Suppose A is any 3 × 3 non-singular matrix and (A – 3I) (A – 5I) = O, where $I = I_3$ and $O = O_3$.

If $\alpha A + \beta A^{-1} = 4I$, then $\alpha + \beta$ is equal to

(1*) 8 (2) 7 (3) 13 (4) 12

Sol. $A^2 - 8A + 15I = 0$

$A^2 = 8A - 15I$

$A = 8I - 15A^{-1}$

$\alpha = \frac{1}{2}, \beta = \frac{+15}{2}$

$\alpha + \beta = 8$

56. If the mean of the data : 7, 8, 9, 7, 8, 7, λ, 8 is 8, then the variance of this data is

(1) 2 (2) $\frac{9}{8}$ (3*) 1 (4) $\frac{7}{8}$

Sol. $\pi = \frac{7+8+9+7+8+7+\lambda+8}{8} = 8 \qquad [\lambda = 10]$

x	f	x_i^2	$f x_i$
7	3	49	21
8	3	64	24
9	1	81	9
10	1	100	10

$\sigma^2 = \frac{1}{n}\Sigma f$

Aliter: $\bar{x} = 8 = \frac{54+\lambda}{8}$

$\lambda = 10$

$$\sigma^2 = \frac{\Sigma(x_i - \bar{x})^2}{N}$$

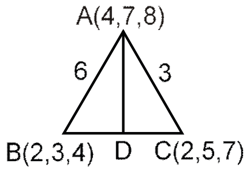
$$\sigma^2 = \frac{1+0+1+1+0+1+4+0}{8} = 1$$

57. If the position vectors of the vertices A, B and C of a ΔABC are respectively $4\hat{i}+7\hat{j}+8\hat{k}$, $2\hat{i}+3\hat{j}+4\hat{k}$ and $2\hat{i}+5\hat{j}+7\hat{k}$, then the position vector of the point, where the bisector of $\angle$A meets BC is

(1) $\frac{1}{4}\left(8\hat{i}+14\hat{j}+19\hat{k}\right)$ (2) $\frac{1}{2}\left(4\hat{i}+8\hat{j}+11\hat{k}\right)$ (3*) $\frac{1}{3}\left(6\hat{i}+13\hat{j}+18\hat{k}\right)$ (4) $\frac{1}{3}\left(6\hat{i}+11\hat{j}+15\hat{k}\right)$

Sol. AB : AC = BD : CD = 2 : 1

$D\left(2, \frac{13}{3}, 6\right)$



58. A normal to the hyperbola, $4x^2 - 9y^2 = 36$ meets the co-ordinate axes x and y at A and B, respectively. If the parallelogram OABP (O being the origin) is formed, then the locus of P is

(1) $4x^2 - 9y^2 = 121$ (2) $4x^2 + 9y^2 = 121$ (3*) $9x^2 - 4y^2 = 169$ (4) $9x^2 + 4y^2 = 169$

Sol. Let P(h, k)

Equation of normal at (x, y)

$$\frac{9x}{x_1} - \frac{4y}{y_1} = 13$$

$$A\left(\frac{13x_1}{9}, 0\right) \quad B\left(0, \frac{-13}{4}y_1\right)$$

Now mid point of OA = mid point of PB

$$\frac{13x_1}{9\times 2} = \frac{h}{2} \quad \frac{-13}{4}\frac{y_1}{(2)} = \frac{k}{2}$$

$$\Rightarrow \quad \boxed{x_1 = \frac{9h}{13}} \quad \boxed{y_1 = \frac{-4k}{13}}$$

Now $(x_1 y_1)$ lie on hyperbola

$$\frac{4\times 81}{169}h^2 - \frac{9\times 16}{169}k^2 = 36$$

$\Rightarrow$ $9h^2 - 4k^2 = 169$

59. A plane bisects the line segment joining the points (1, 2, 3) and (–3, 4, 5) at right angles. Then this plane also passes through the point

(1*) (–3, 2, 1) (2) (3, 2, 1) (3) (–1, 2, 3) (4) (1, 2, –3)

Sol. d, rs of normal = d, rs of AB

Mid point of A(1, 2, 3) and B(–3, 4, 5) lie on plane. M(–1, 3, 4)

Equation of plane $4(x + 1) - 2(y - 3) - 2(x - 4) = 0$

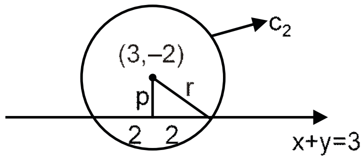
$$\boxed{2x - y - z + 9 = 0}$$

60. The tangent to the circle $C_1 : x^2 + y^2 - 2x - 1 = 0$ at the point (2, 1) cuts off a chord of length 4 from a circle C_2 whose centre is (3, –2). The radius of C_2 is

(1) 2 (2) 3 (3*) $\sqrt{6}$ (4) $\sqrt{2}$

Sol. Equation of tangent $2x + y - (x + 2) - 1 = 0$

$$\boxed{x + y - 3 = 0}$$



$$P = \frac{|3 - 2 - 3|}{\sqrt{2}} = \sqrt{2}$$

$$r^2 = P^2 + 4$$

$$\boxed{r = \sqrt{6}}$$

PART-C-PHYSICS

61. An unstable heavy nucleus at rest breaks into two nuclei which move away with velocities in the ratio of 8 : 27. The ratio of the radii of the nuclei (assumed to be spherical) is :

(1) 4 : 9 (2) 2 : 3 (3) 8 : 27 (4*) 3 : 2

Sol. $m_1v_1 = m_2v_2$

$\therefore \frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{27}{8}$

$\therefore \frac{r_1^3}{r_2^3} = \frac{27}{8}$

$\therefore \frac{r_1}{r_2} = \frac{3}{2}$

62. 5 beats/second are heard when a tuning fork is sounded with a sonometer wire under tension, when the length of the sonometer wire is either 0.95 m or 1 m. The frequency of the fork will be :

(1*) 195 Hz (2) 251 Hz (3) 300 Hz (4) 150 Hz

Sol. $\frac{v}{2\ell_1} - f = 5$

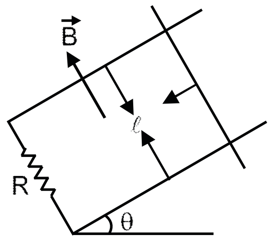
$f - \frac{v}{2\ell_2} = 5$

$\therefore \frac{\ell_2}{\ell_1} = \frac{f+5}{f-5}; \ell_1 = 0.95 \text{ m}$

$\ell_2 = 1 \text{ m}$

$\therefore f = 195 \text{ Hz}$

63. A copper rod of mass m slides under gravity on two smooth parallel rails, with separation 1 and set at an angle of θ with the horizontal. At the bottom, rails are joined by a resistance R. There is a uniform magnetic field B normal to the plane of the rails, as shown in the figure. The terminal speed of the copper rod is :



(1) $\frac{mgR\cos\theta}{B^2\ell^2}$ (2*) $\frac{mgR\sin\theta}{B^2\ell^2}$

(3) $\frac{mgR\tan\theta}{B^2\ell^2}$ (4) $\frac{mgR\cot\theta}{B^2\ell^2}$ac

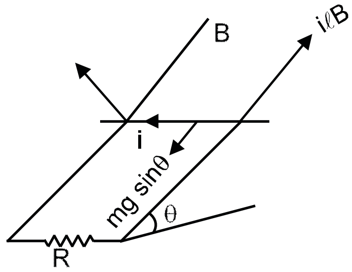
Sol. When terminal velocity is acquired net force on the rod becomes zero.

$i\ell B = mg\sin\theta$

$\therefore \dfrac{vB^2\ell^2}{R} = mg\sin\theta$

$\therefore v = \dfrac{mg\,R\sin\theta}{B^2\ell^2}$

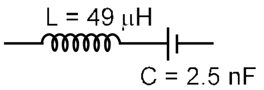
$= \dfrac{mg\,R\sin\theta}{B^2\ell^2}$



64. The carrier frequency of a transmitter is provided by a tank circuit of a coil of inductance 49μH and a capacitance of 2.5 nF. It is modulated by an audio signal of 12 kHz. The frequency range occupied by the side bands is :

(1) 13482 kHz – 13494 kHz
(2) 63 kHz - 75 kHz
(3) 18 kHz – 30 kHz
(4*) 442 kHz – 466 kHz

Ans.



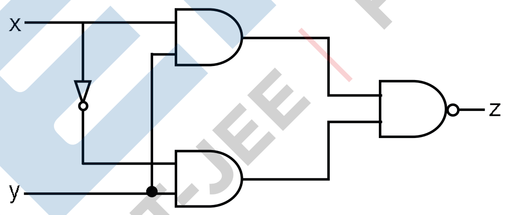
$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{49\times10^{-6}\times2.5\times10^{-9}}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{7\times5\times10^{-8}\times2\times\frac{22}{7}} = \frac{10^7}{22}$$

= 454.5 kHz

(454.5 ± 12) kHz

65. Truth table for the following digital circuit will be



(1)

x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

(2)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	0

(3)

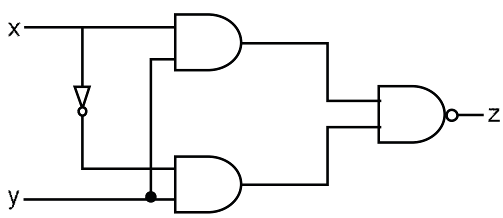
x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

(4*)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	1

Sol.

x	y	$\bar{x}$	A	B	z
0	0	1	0	0	1
0	1	1	0	1	1
1	0	0	0	1	1
1	1	0	1	0	1



66. A parallel plate capacitor with area 200 cm^2 and separation between the plates 1.5 cm, is connected across a battery of emf V. If the force of attraction between the plates is 25 × 10^{-6} N, the value of V is approximately : $\left(\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}\right)$

(1) 100 V (2) 150 V (3) 300 V (4*) 250 V

Sol. $F = \frac{Q^2}{2A\epsilon_0} = \frac{c^2v^2}{2A\epsilon_0}$

$= \frac{\epsilon_0^2 A^2}{d^2} \times \frac{v^2}{2A\epsilon_0}$

$= \frac{\epsilon_0 Av^2}{2d^2}$

$25 \times 10^{-6} = \frac{8.85 \times 10^{-12} \times 2 \times 10^{-2} \times v^2}{2 \times 2.25 \times 10^{-4}}$

v = 250 v

67. The value closest to the thermal velocity of a Helium atom at room temperature (300 K) in ms^{-1} is :

[k_B = 1.4 × 10^{-23} J/K ; m_{He} = 7 × 10^{-27} kg]

(1) 1.3 × 10^5 (2) 1.3 × 10^4 (3*) 1.3 × 10^3 (4) 1.3 × 10^2

Sol. $v_{rms} = \sqrt{\frac{3 \times 1.4 \times 10^{-23} \times 300}{7 \times 10^{-27}}}$

$= 3 \times 2\sqrt{5} \times 10^2$

= 13.42 × 10^2

= 1.3 × 10^3

68. A convergent doublet of separated lenses, corrected for spherical aberration, has resultant focal length of 10 cm. The separation between the two lenses is 2 cm. The focal lengths of the component lenses are:

(1*) 18 cm, 20 cm (2) 12 cm, 14 cm (3) 10 cm, 12 cm (4) 16 cm, 18 cm

Sol. $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

$\frac{1}{10} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{2}{f_1 f_2}$

First data satisfies above relation.

69. Two simple harmonic motions, as shown below, are at right angles. They are combined to form Lissajous figures.

x(t) = A sin (at + δ)

y(t) = B sin (bt)

Identify the correct match below

Parameters Curve Parameters Curve

(1) A ≠ B, a = b ; $\delta = 0$ Parabola (2) A = B, a = 2b ; $\delta = \frac{\pi}{2}$ Circle

(3) A = B, a = b ; $\delta = \frac{\pi}{2}$ Line (4*) A ≠ B, a = b ; $\delta = \frac{\pi}{2}$ Ellipse

Sol. (1) $x = 4\left(1 - \frac{2y}{b^2}\right)$ (parabola)

(2) x = A sin at

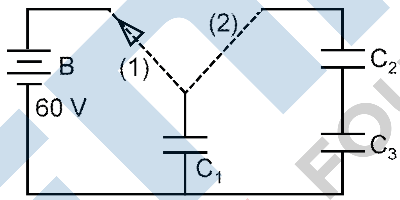
y = B sin at

(3) $\frac{x}{A} = \cos at$

$\frac{y}{B} = \sin at$

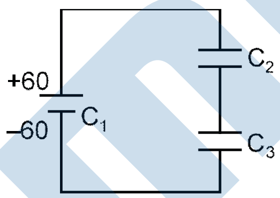
$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$

70. A capacitor C_1 = 1.0 μF is charged up to a voltage V = 60 V by connecting it to battery B through switch (1). Now C_1 is disconnected from battery and connected to a circuit consisting of two uncharged capacitors C_2 = 3.0 μF and C_3 = 6.0 μF through switch (2), as shown in figure. The sum of final charges on C_2 and C_3 is :



(1) 20μC (2) 54μC (3) 36μC (4*) 40μC

Sol. Q_1 = 60 μC



Equivalent capacitance of C_2 and C_3

$$C' = \frac{C_2C_3}{C_2 + C_3} = \frac{3 \times 6}{3 \times 6}\mu F = 2\mu F$$

Common potential difference to C_1 and C' combined.

$$v' = \frac{C_1v_1 + 0}{C_1 + C'} = \frac{60}{1+2} = 3 \text{ volts}$$

Charge of C_2, C_3 system = C'v = 2 × 20 = 40 μC

71. At the centre of a fixed large circular coil of radius R, a much smaller circular coil of radius r is placed. The two coils are concentric and are in the same plane. The larger coil carries a current I. The smaller coil is set to rotate with a constant angular velocity ω about an axis along their common diameter. Calculate the emf induced in the smaller coil after a time t of its start of rotation.

(1) $\frac{\mu_0 I}{4R}\omega\pi r^2 \sin\omega t$ (2) $\frac{\mu_0 I}{4R}\omega r^2 \sin\omega t$

(3*) $\frac{\mu_0 I}{2R}\omega\pi r^2 \sin\omega t$ (4) $\frac{\mu_0 I}{2R}\omega r^2 \sin\omega t$

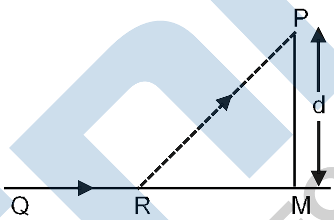
Sol. Flux ϕ = BA

$$\pi = \frac{\mu_0 I}{2R}\pi r^2 \cos\theta$$

$$\text{e.n.f. } e = \frac{-d\phi}{dl} = \frac{\mu_0 I}{2R}\pi\omega\cos\theta$$

$$\text{So } \frac{\mu_0 I}{2R}\omega\pi r^2 \cos\theta$$

72. A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from the highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM, so that the time taken to reach P is minimum ?



(1*) $\frac{d}{\sqrt{3}}$ (2) $\frac{d}{2}$ (3) d (4) $\frac{d}{\sqrt{2}}$

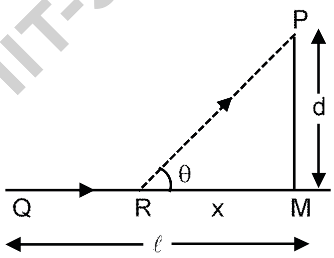
Sol. $t = \frac{\ell - x}{v} + \frac{\sqrt{x^2 + d^2}}{v/2}$

$$\frac{dt}{dx} = \frac{1}{v}(0-1) + \frac{2}{v}\frac{2x}{2\sqrt{x^2+d^2}}$$

$$\cos\theta = \frac{1}{2}$$

θ = 60°

$$x = \frac{d}{\sqrt{3}}$$



73. A solid ball of radius R has a charge density ρ given by $r = r_0\left(1 - \frac{r}{R}\right)$ for $0 \le r \le R$. The electric field outside the ball is :

(1) $\frac{\rho_0 R^3}{\epsilon_0 r^2}$ (2) $\frac{3\rho_0 R^3}{4\epsilon_0 r^2}$ (3*) $\frac{\rho_0 R^3}{12\epsilon_0 r^2}$ (4) $\frac{4\rho_0 R^3}{3\epsilon_0 r^2}$

Sol. $E \times 4\pi r^2 = \int_0^R \rho_0 \left(1 - \frac{r}{R}\right) 4\pi r^2 dr \times \frac{1}{\epsilon_0}$

$= \frac{\rho_0 4\pi}{\epsilon_0} \times \left[\frac{R^3}{3} - \frac{R^3}{4}\right]$

$\therefore Er^2 = \rho_0 \times \frac{R^3}{12 \epsilon_0}$

$\therefore E = \frac{\rho_0 R^3}{12 \epsilon_0 r^2}$

74. A body takes 10 minutes to cool from 60°C to 50°C. The temperature of surroundings is constant at 25°C. Then, the temperature of the body after next 10 minutes will be approximately :

(1) 41°C (2) 45°C (3*) 43°C (4) 47°C

Sol. $\frac{60-50}{10} = k[55-25]$

$\frac{60-\theta}{20} = k\left[\frac{60+\theta}{2} - 25\right]$

$\frac{20}{60-\theta} = \frac{30}{30 + \frac{\theta}{2} - 25}$

$\therefore \frac{2}{60-\theta} = \frac{3}{5 + \frac{\theta}{2}}$

$\therefore$ 10 + θ = 180 – 3θ

$\therefore$ 4θ = 170

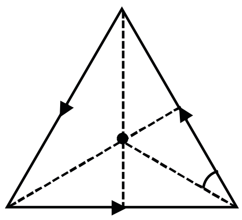
$= \frac{85}{2} = 42.5°C$

75. A current of 1 A is flowing on the sides of an equilateral triangle of side 4.5 × 10^{-2} m. The magnetic field at the centre of the triangle will be :

(1) 2 × 10^{-5} Wb / m^2 (2) Zero (3) 8 × 10^{-5} Wb/m^2 (4*) 4 × 10^{-5} Wb / m^2

Sol. $\frac{\mu_0 i}{4\pi d}(\cos\alpha + \cos\beta) \times 3$

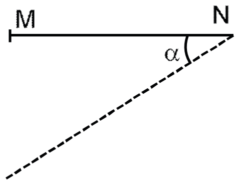
$= \left(\frac{4\pi \times 10^{-7} \times 1 \times 2\sqrt{3} \times \sqrt{3}}{4\pi \times 4.5 \times 10^{-2}}\right) \times 3$



$= \frac{2 \times 10^{-5} \times 2 \times 3 \times 3}{4.5 \times 2}$

= 4 × 10^{-5} Wb/m^2

76. A thin rod MN, free to rotate in the vertical plane about the fixed end N, is held horizontal. When the end M is released the speed of this end, when the rod makes an angle α with the horizontal, will be proportional to : (see figure)



(1) $\sin\alpha$ (2) $\cos\alpha$ (3*) $\sqrt{\sin\alpha}$ (4) $\sqrt{\cos\alpha}$

Sol. $\frac{1}{2}\times\frac{1}{3}m\ell^2\times\omega^2 = mg\frac{\ell}{2}\sin\alpha$

$\therefore \omega = \sqrt{\frac{3g\sin\alpha}{\ell}}$

$\therefore v = \omega\ell = \sqrt{3g\ell\sin\alpha}$

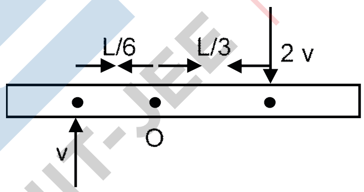
77. A constant voltage is applied between two ends of a metallic wire. If the length is halved and the radius of the wire is doubled, the rate of heat developed in the wire will be :

(1) Doubled (2) Unchanged (3*) Increased 8 times (4) Halved

Sol. $R = \frac{\rho\ell}{A}, R' = \frac{\rho(\ell/2)}{4A}$

$P = \frac{v^2}{R}, P' = \frac{v^2}{R'} = \frac{V^2}{R'} = \frac{8v^2}{R} = 8P$

78. A thin uniform bar of length L and mass 8 m lies on a smooth horizontal table. Two point masses m and 2 m are moving in the same horizontal plane from opposite sides of the bar with speeds 2 v and v respectively. The masses stick to the bar after collision at a distance $\frac{L}{3}$ and $\frac{L}{6}$ respectively from the centre of the bar. If the bar starts rotating about its center of mass as a result of collision, the angular speed of the bar will be :



(1*) $\frac{6v}{5L}$ (2) $\frac{3v}{5L}$ (3) $\frac{v}{5L}$ (4) $\frac{v}{6L}$

Sol. After collision center of mass remain at O.

$m_1r_1 = m_2r_2$

$2m\frac{L}{6} = mL/3$

Using conservation of angular momentum

$m(2v)\frac{L}{3} + 2mv\frac{L}{6} = I\omega \quad ......(1)$

when $I = 8m\frac{L^2}{12} + 2m\left(\frac{L}{6}\right)^2 + m\left(\frac{L}{3}\right)^2$

$= \frac{5}{6} mL^2$

From (1) $\left(\frac{2}{3}+\frac{2}{6}\right) mvL = \frac{5}{6} mL^2\omega$

or $\omega = \frac{5}{6}\frac{v}{\ell}$

79. A body of mass 2 kg slides down with an acceleration of 3 m/s^2 on a rough inclined plane having a slope of 30°. The external force required to take the same body up the plane with the same acceleration will be : (g = 10 m/s^2)

(1) 14 N (2*) 20 N (3) 4 N (4) 6 N

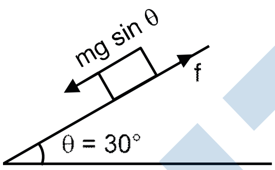
Sol. For downward motion

$mg \sin\theta - f = ma$

or $f = mg \sin\theta - ma$

$= 2 \times 10 \times ½ - 2 \times 3$

$= 4$ N



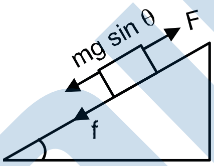
For upward motion

$F - mg \sin\theta - f = ma$

or $F = mg \sin\theta + F + ma$

$= 2 \times 10 \times ½ + 4 + 2 \times 3$

$= 10 + 4 + 6 = 20$ N



80. Muon (μ^-) is a negatively charged (|q| = |e|) particle with a mass $m_\mu = 200\ m_e$, where m_e is the mass of the electron and e is the electronic charge. If μ^- is bound to a proton to form a hydrogen like atom, identify the correct statements.

(A) Radius of the muonic orbit is 200 times smaller than that of the electron.

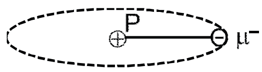
(B) The speed of the μ^- in the nth orbit is $\frac{1}{200}$ times that of the electron in the nth orbit.

(C) The ionization energy of muonic atom is 200 times more than that of an hydrogen atom.

(D) The momentum of the muon in the nth orbit is 200 times more than that of the electron.

(1) (A), (B), (D) (2) (B), (D) (3*) (A), (C), (D) (4) (C), (D)

Sol. $r \propto \frac{1}{m}$



v is independent of m

Energy α m

81. A proton of mass m collides elastically with a particle of unknown mass at rest. After the collision, the proton and the unknown particle are seen moving at an angle of 90° with respect to each other. The mass of unknown particle is :

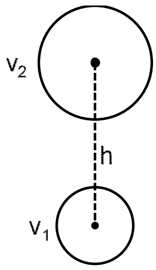
(1) $\frac{m}{\sqrt{3}}$ (2) 2m (3) $\frac{m}{2}$ (4*) m

Sol. Elastic collision between two masses leads to such a situation.

82. When an air bubble of radius r rises from the bottom to the surface of a lake, its radius becomes $\frac{5r}{4}$. Taking the atmospheric pressure to be equal to 10 m height of water column, the depth of the lake would approximately be (ignore the surface tension and the effect of temperature) :

(1) 8.7 m (2) 10.5 m (3*) 9.5 m (4) 11.2 m

Sol. $P_1v_1 = P_2v_2$



$\therefore (P_0 + h\rho_w g)\frac{4\pi}{3}r^3$

$= P_0 \times \frac{4}{3}\pi\left(\frac{5r}{4}\right)^3$

$\therefore (10 + h)\rho_w g = 10\rho_w g \times \frac{125}{64}$

$\therefore$ 640 + 64 h = 1250 $\therefore$ 64 h = 610

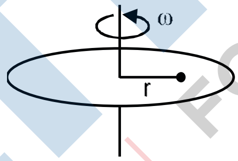
$\therefore$ h = 9.5 m

83. A disc rotates about its axis of symmetry in a horizontal plane at a steady rate of 3.5 revolutions per second. A coin placed at a distance of 1.25 cm from the axis of rotation remains at rest on the disc. The coefficient of friction between the coin and the disc is : ($g = 10\ m/s^2$)

(1) 0.6 (2) 0.5 (3*) 0.7 (4) 0.3

Sol. $\omega = 3.5 \times 2\pi$ rad/s

$= 7\pi$ rad/s



In the frame of disc : $m\omega^2 r = \mu mg$

$\therefore \mu = \frac{\omega^2 r}{g} = \frac{49\pi^2 \times 1.25 \times 10^{-2}}{10}$

 $49 \times 1.25 \times 10^{-2}$

= 0.60

84. Two Carnot engines A and B are operated in series. Engine A receives heat from a reservoir at 600 K and rejects heat to a reservoir at temperature T. Engine B receives heat rejected by engine A and in turn rejects it to a reservoir at 100 K. If the efficiencies of the two engines A and B are represented by η_A and η_B, respectively, then what is the value of $\frac{\eta_B}{\eta_A}$?

(1) $\frac{5}{12}$ (2) $\frac{12}{5}$ (3*) $\frac{12}{7}$ (4) $\frac{7}{12}$

Sol. $\eta_A = 1 - \dfrac{T}{600}$

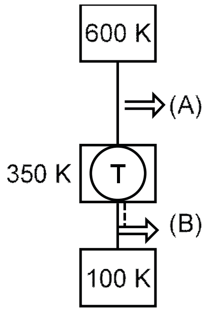
$\eta_B = 1 - \dfrac{100}{T}$

Assuming T = 350 K (in series)

$\dfrac{\eta_B}{\eta_A} = \dfrac{T-100}{T} \times \dfrac{600}{600-T}$

$= \dfrac{250}{350} \times \dfrac{600}{250}$

= 12/7



85. The characteristic distance at which quantum gravitational effects are significant, the Planck length, can be determined from a suitable combination of the fundamental physical constants G, $\hbar$ and c. Which of the following correctly gives the Planck length ?

(1) $G\hbar^2c^3$ (2) $G^{1/2}\hbar^2c$ (3) $G^2\hbar c$ (4*) $\left(\dfrac{G\hbar}{c^3}\right)^{1/2}$

Sol. ℓ = f (Ghc)

$[\ell] = [M^{-1}L^3T^{-2}]^x\,[ML^2T^{-1}]^y\,[LT^{-1}]^z$

$= [M^{-x+y}]\,[L^{2x+2y+z}]\,[T^{-2x-y-z}]$

$$-x+y=0 \;\left|\; \begin{array}{l} -2x-y-z=0 \\ \therefore -3x-z=0 \\ \therefore z=-3x \end{array} \right|\; \begin{array}{l} 3x+2y+z=1 \\ 3x+2x-3x=0 \\ \therefore x=\dfrac{1}{2}=y \end{array}$$

z = –3/2

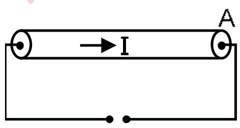
$\therefore \ell = \left(G^{1/2}h^{1/2}c^{-3/2}\right) = \sqrt{\dfrac{Gh}{c^3}}$

86. A copper rod of cross-sectional area A carries a uniform current I through it. At temperature T, if the volume charge density of the rod is ρ, how long will the charges take to travel a distance d?

(1) $\dfrac{\rho dA}{IT}$ (2*) $\dfrac{\rho dA}{I}$ (3) $\dfrac{2\rho dA}{I}$ (4) $\dfrac{2\rho dA}{IT}$

Sol. $I = neAv_d = \rho A v_d$

$\therefore t = \dfrac{d}{v_d} = \dfrac{d\rho A}{I}$



87. If the de Broglie wavelengths associated with a proton and an α-particle are equal, then the ratio of velocities of the proton and the α-particle will be :

(1) 1 : 4 (2*) 4 : 1 (3) 1 : 2 (4) 2 : 1

Sol. $\lambda = \dfrac{h}{mv}$

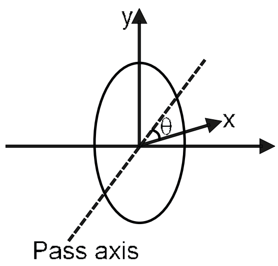
λ is same so

mv is also same

$m_\alpha v_\alpha = m_p v_p$

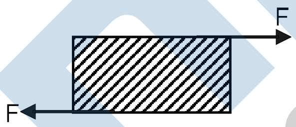
$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4}{1} = 4$

88. A plane polarized light is incident on a polariser with its pass axis making angle θ with x-axis, as shown in the figure. At four different values of θ, θ = 8°, 38°, 188° and 218°, the observed intensities are same. What is the angle between the direction of polarization and x-axis?



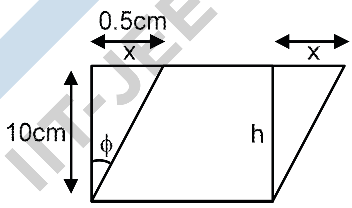
(1) 98° (2) 128° (3) 45° (4*) 203°

89. As shown in the figure, forces of 10^5 N each are applied in opposite directions, on the upper and lower faces of a cube of side 10 cm, shifting the upper face parallel to itself by 0.5 cm. If the side of another cube of the same material is 20 cm. then under similar conditions as above, the displacement will be :



(1) 0.75 cm (2) 0.37 cm (3*) 0.25 cm (4) 1.00 cm

Sol. $\eta = \frac{F/A}{\phi} = \frac{Fh}{Ax}$



$\therefore x = \frac{Fh}{A\eta}$

$\therefore \frac{x_1}{x_2} = \frac{h_1}{h_2} \times \frac{A_2}{A_1}$

$= \frac{10}{20} \times \frac{400}{100}$

$= 2$

$\therefore \frac{x_2}{x_1} = \frac{1}{2}$

$\therefore x_2 = \frac{x_1}{2} = \frac{0.5}{2} = 0.25$ cm

90. A plane polarized monochromatic EM wave is traveling in vacuum along z direction such that at $t = t_1$ it is found that the electric field is zero at a spatial point z_1. The next zero that occurs in its neighborhood is at z_2. The frequency of the electromagnetic wave is :

(1) $\frac{3\times10^8}{|z_2-z_1|}$ (2) $\frac{1}{t_1+\frac{|z_2-z_1|}{3\times10^8}}$ (3) $\frac{6\times10^8}{|z_2-z_1|}$ (4*) $\frac{1.5\times10^8}{|z_2-z_1|}$

Sol. $v=\frac{\nu}{\lambda}$

Here $\frac{\lambda}{2}=|z_1-z_2|$

$v=\frac{\nu}{2|(z_2-z_1)|}$

$=\frac{3\times10^8}{2|(z_2-z_1)|}$

$=\frac{1.5\times10^8}{|z_2-z_1|}$